ANALYTICAL APPROXIMATIONS

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PREFACE

This is a draft of a report which is being circulated for information and comment. We hope to make it a chapter of a book titled Military Planning In An Uncertain World, and would appreciate any comments, criticism, ideas, and examples that readers may have. This draft began as a transcript of an informal talk and, despite some rewriting, it probably still suffers (like many such talks) from being "fashionable." We are aware that it has a number of other weaknesses and assume there are still others of which we are not aware. We hope to give it a thoughtful and leisurely review but are deferring this until we get some outside criticism.

A table of contents is given on the next page to show the relation of this chapter to the rest of the book. The chapter may not be quite self-contained as a paper, as it occasionally refers to other chapters; but we trust this will be understood or overlooked.

A more complete introduction and list of acknowledgements are given in RM-1829-1.

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Analytical Approximation

Chi-Square Integral: To better than .0018 over $0 \leq \chi^2 \leq m$ and $2 \leq m < \infty$, m being considered a continuous parameter,

$$F_m(\chi^2) = \frac{1}{2\Gamma\left(\frac{m}{2}\right)} \int_0^{\chi^2} \left(\frac{t^2}{2}\right)^{\frac{m}{2}-1} e^{-\frac{1}{2}t^2} d(t^2)$$

$$\doteq \frac{A}{\left[1+a_1\eta+a_2\eta^2+a_3\eta^3\right]^4}$$

$$\eta = \sqrt{\frac{m}{2}\ln\left(\frac{m}{\chi^2}\right)}$$

$$A = .5 + .133\sqrt{\frac{2}{m}}$$

$$a_1 = .209 - \frac{.138}{\sqrt{2m+1}}$$

$$a_2 = .061 + .030\sqrt{\frac{2}{m}} - .043\left(\frac{2}{m}\right)$$

$$a_3 = .062 - \frac{.173}{\sqrt{m+6}}$$

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